

---

Christoph Dürr  
CNRS, CMM  
University of Chile  
`christoph.durr@CNRS.fr`

---

## Report on the PhD manuscript *The Price of Delays in Online Algorithms* by Artur Kraska

This manuscript contains excellent results, which are really well written. Turning the last page leaves the reader with the impression of a solid PhD thesis. To be honest, I am deeply impressed by the overall work.

Traditionally the area of online algorithms deals with sequences of requests which have to be served immediately and irrevocably. Each serving generates some cost. The total cost could have been optimized if one knew the whole request sequence in advance. Normalizing the cost of an online algorithm with the offline optimum, leads to the so-called *competitive ratio*. Now the variant to which Artur contributed allows the algorithm to delay the request services. This generates an additional delay cost, but the hope is that better decisions can be made when services are delayed.

The first chapter summarizes in 10 pages all techniques one needs to know in order to follow the rest of the manuscript. It restricts to the minimum necessary and hence keeps this part easy and not imposing. Chapter two presents an algorithm called *Mimic*, which has great performances for the travelling repair-person problem, the Dial-a-Ride problem and the Unrelated Machines Scheduling problem, possibly with precedence constraints. These are well studied problems, and I am extremely impressed by the improvement of the competitive ratio on these problems. In particular Artur improves over a 25 years old result. It is really appreciable that a unifying algorithm is presented here, explaining well the different ingredients. Roughly their algorithm follows *Plan-And-Commit*, which was studied earlier, but solving some subtle problems. It works in phases of geometrically increasing lengths, and in each phase it executes a minimizing the completion time, adding a penalty for not executed jobs, which reflects their delay to the next phase. The analysis uses the primal-dual framework, and to deal with the continuous world of randomized algorithms, the limit of a discretization of the random space is considered. The analysis represents quite some work with the linear program.

In the third chapter, Artur describes his contribution to *Online Service with Delay*. This problem has been studied quite recently with a first result dating from 2017, and using hierarchically separated trees. But the presented result is for a simple and elementary metric, consisting of  $n$  equidistant points on the line. The principle of their algorithm called *Bucket* is quite simple and follows the lessons learned from the ski rental problem and from the cow path problem. It works in phases, subdivided into sub-phases. In a waiting sub-phase the server does nothing but waiting until for some group of requests, the waiting cost balances in some sense their serving cost, then it serves this group plus some more in the subsequent serving sub-phase. Requests are grouped in buckets according to their distance to the current server position, rounded to the next power of 2. The analysis uses a subtle charging scheme.

The second last, but last technical chapter concerns *Online Matching with Delays*. This is a variant of the well studied bipartite graph min cost perfect matching problem, where vertices on the right arrive online. There are very good motivations to this problem, such as matching players of similar rank in an online gaming platform, or matching donors and patients for kidney transplants. Vertices arrive online at some arrival time, with some polarization, and a position in a metric space. Artur achieved a  $O(m)$ -competitive algorithm using the primal-dual approach, improving previous results from 2017. Here  $m$  is the number of requests. Meanwhile a follow-up work improved their result, testifying the scientific competition on these problems. There were many technical difficulties to overcome when adapting the moat-growing framework to this online problem. A great improvement is due to the fact that the authors do avoid embedding the metric

in a tree-metric, and as a result present an algorithm which can deal with metric spaces that are discovered online with the arriving requests.

Besides personal and published work, the manuscript contains some additional interesting material: a mistake in a previously published paper, and some unsuccessful attempts together with their counter-examples. The manuscript ends with a last chapter summarizing the presented work and indicating future research directions. Artur Kraska published 6 papers, in well recognized conferences, the latest being ICALP. Once again, this PhD manuscript describes excellent work and describes it well. I am very in favour of giving him the doctoral degree, and suggest to nominate the thesis for a prestigious doctoral dissertation award.

Christoph Dürr – Santiago, January 5th, 2022

A handwritten signature in blue ink, appearing to read "Christoph Dürr", with a long horizontal flourish underneath.